Stability Metrics, Design Methods, and a Variable Stiffness Actuator for Use in Passive-Dynamic Robots

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Abstract

A metric of passive-dynamic stability, and a design method for passive-dynamic machines are presented. A model of a passive-dynamic robot which is hypothesized to be capable of walking and running is presented. This model has been selected to match closely with a physical robot currently in construction, for the purpose of furthering the state-of-the-art in legged machines which locomote efficiently based on passive-dynamic principles. A novel actuator called the Variable Stiffness Series Elastic Actuator (VSSEA) is also introduced. Its design and advantages are briefly discussed.

1. What are the most important questions addressed in this research?

- How do we quantify stability for passive-dynamic robots? How do we compare the stability of two passive dynamic robots with different morphologies?
- How do we get a robot to move the way we want without specifying an exact kinematic path?
- How do we maintain the energetic efficiency and stability of a passive-dynamic robot while extending its practical utility via an active control system?

2. Passive-Dynamic Robot Design Concept

The following is an outline of the proposed design method for passive-dynamic robots. There are several important assumptions made here: that mechanical and control dynamics can be designed fairly independently, that stable walking/running motions down an inclined plane are sufficiently similar or relevant to any walking/running motion that they can be considered an optimal or desirable motion, and that this motion is energetically efficient.
The proposed design procedure follows.

1. *Mechanical Stability.* Design mechanics of robot for stable motion down a shallow slope. Optimize speed/efficiency/stability tradeoffs using appropriate metrics. Possible stability metrics are defined in section 3.
   (a) If no limit cycle exists for the model, adjust parameters until one exists, or add a very simple control rule to create a minimal limit cycle.
   (b) We ignore the possiblity of stabilizing a slightly unstable walker at this step. We assume that good robot designs are naturally stable, much like good aircraft designs are naturally stable.
2. *Oscillation-Sustaining Control.* Use an appropriate 'virtual slope' control rule to sustain walking/running motions for an arbitrary real slope.
   (a) It seems plausible that a variety of control rules would be sufficient to sustain forward motion, including CPGs, virtual gravity, constant-energy control, etc. The best rules will do only positive work and have little or no negative effect on overall stability.
   (b) Forward speed is thus controlled by adjusting the virtual gravity vector.
   (c) For some passive-dynamic models, the transition from walking to running is presumed to occur naturally as a result of a change in virtual slope.
3. *Additional Stability Control.* Add a control rule to arbitrarily improve stability in exchange for some energy cost.
   (a) Control must not overwhelm the passive dynamics. For this task, force/energy control is far more suitable than high-gain position control.
   (b) Control for a compass biped is 5 dimensional: 4 state variables plus the virtual slope. This is a difficult problem, especially because analytical control is difficult and we are forced to map out stable region numerically.
   (c) The instant immediately after a collision may be a good choice to use as a 'reference' slice, reducing dimensionality to 4. (2 velocities + inter-leg angle + virtual slope)
   (d) The job of the control system is to ensure that the state of the robot will stay in the stable domain around the limit cycle, by controlling the above 4 quantities.

3. Passive-Dynamic Stability Metrics

If engineering is at its root simply a process of optimization of some task, then perhaps most important choice in any engineering task is the choice of metrics: what quantity do we optimize? Although the mathematical concept of stability is a fairly well understood idea, moving from the question "Is it stable?" to the more interesting "How stable is it?" is a difficult transition, especially for mechanical systems.
In classic linear control schemes, metrics like settling time, step response, frequency response, phase and gain margins are very popular among engineers. We thus make an analogy between typical linear control engineering metrics and some suggested quantities which may have relevance in the problem domain of passive dynamic walking.

Engineering Metric	*Passive-Dynamic Metric*
Settling Time	After perturbation, number of steps until within x% of limit cycle.
Noise Margins	Minimum perturbation (from external or control force) which moves state from limit cycle to an unstable region
Frequency Response	Above two measures for many step widths/ slopes

Once suitable metrics have been defined, we can then answer questions like "What foot radius maximizes stability?", "Where should the leg masses be to optimize energetic efficiency?" "What is the tradeoff between energetic efficiency and stability?"

4. What quantity should the control dynamics control? In what manner?

Controlling the joint angles directly is not really compatible with the passive-dynamic philosophy toward system design, because it ultimately results in an obsession with kinematic trajectories.
I believe it would be promising to think of both stability and control in terms of energy, momentum, and impulses. If we consider a dynamic system in terms of energy and impulses,

- A collision can be looked at as an impulse or energy change
- Noise and random disturbances can be seen as impulses or energy changes
- Control inputs can be specified as impulses or energy changes

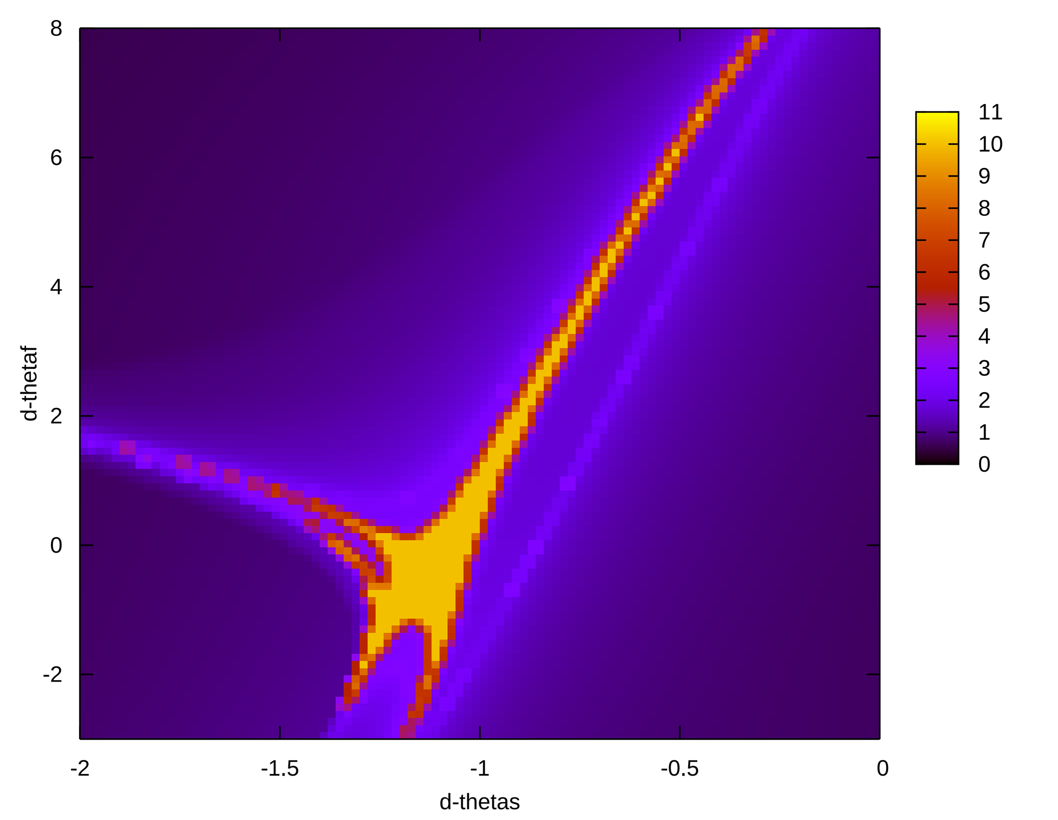

Figure 1: *A 2D slice of the compass biped's limit cycle's attractive basin for a 3 degree slope, taken right after collision. Axis are joint angular velocities.*

5. Simulation

To study the stability of passive-dynamic systems, a simple simulator was written. Completed features of simulation

- Written in LISP, using Matlisp interface to LAPACK, and cl-opengl interface to OpenGL.
- Reasonably Good Performance. Several thousand simulation timesteps possible per second.
- Runge-Kutta 4/5th order numerical integrator.
- Numerical root finding techniques using bisection and Newton-Raphson methods.
- Real time visualization and on-the-fly parameter changing via keypress
- Error checking: numerical integrator tolerances, non-physical checks possible.
- Hybrid systems can be easily modeled as state machines.
- Transitions between states accurate to arbitrary precision.
- Object orientation, encapsulation. Multiple, different models can be run simultaneously.

Incomplete features:

- Improvement of collision accuracy via 1D pseudo Newton-Raphson or Secant method.
- Realtime monitoring with Gnuplot (Currently plotting is not real-time).
- 3D visualizer for limit cycle.

6. Control Using Generalized Momenta

If we consider the graph shown in Fig. 1, we can see the stability of the limit cycle in terms of joint velocities. However, let us consider the instant around collision and ask some important questions.

1. If we were to vary the mechanical parameters of the machine, for example the ratio of hip to leg mass, or length a, has the stability of joint angles really given us any insight into the stability of the system?
2. If we bump or otherwise disturb the machine with an impulse of some amount of energy, is it easy to see how this affects the stability of the machine by looking at this graph?
3. Are the stability of joint angular velocities relevant to models of more complex systems? Can we easily compare models?

The answer to the above questions is somewhat unclear.
However, if we could consider the stability of the robot in terms of energy, we might be answer the above questions in the affirmative. For example, one such metric might be the maximum impulse of energy the robot can accept before falling over. The graph shown in Fig. 2 is the same limit cycle moved into the domain of generalized momenta. Note that the structure of the limit cycle has become more clear and we may even be able to guess the analytic equations of certain curves present here.

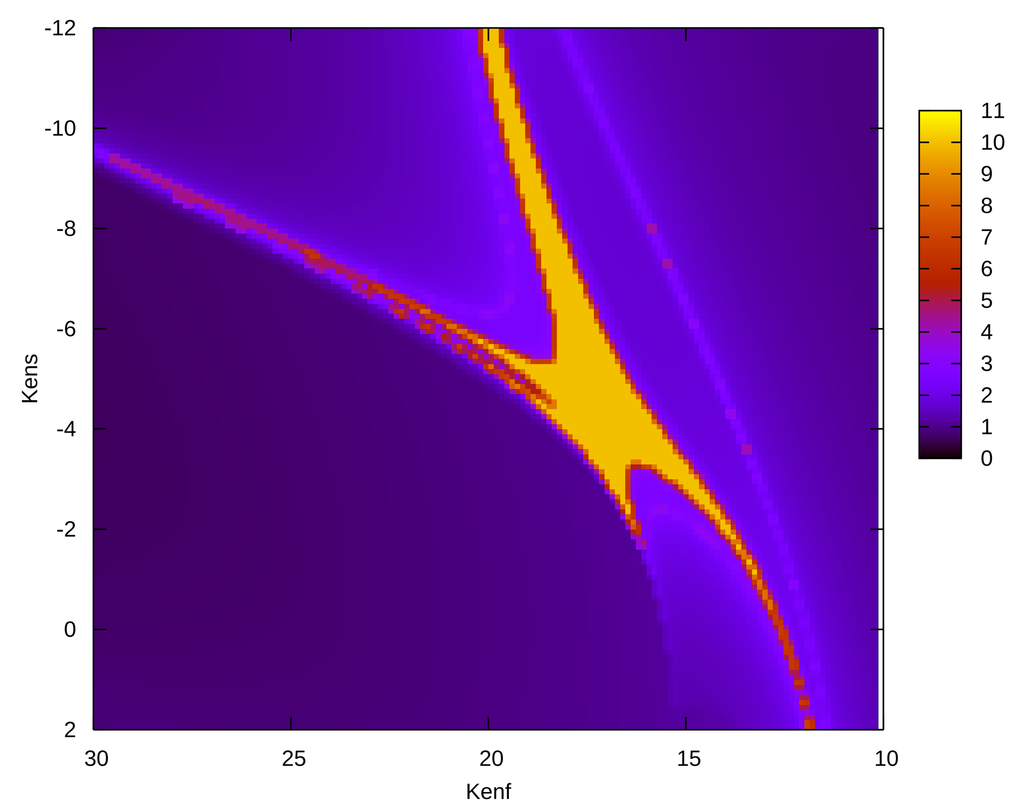

Figure 2: *The same limit cycle presented in Fig. 1. However, these axis are generalized momenta. Graph has been rotated into a new basis (eigenvectors).*

7. What model is under consideration?

- Due to its simplicity, the compass biped will be studied during the theoretical analysis.
- However, an engineering objective of this research is to construct a planar robot that walks and runs efficiently, based on principles of passive-dynamic locomotion.

Prepared for Dynamic Walking III. Mariehamn, Finland. 2007.

- A robot model currently under study that is believed to be capable of walking and running is shown in Fig. 3.

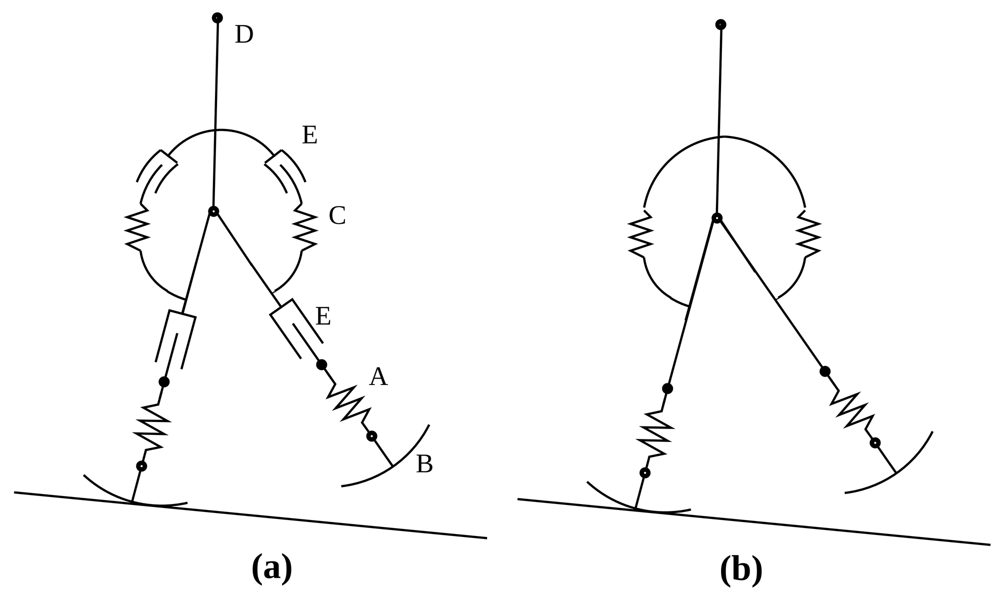

Figure 3: *Model of the robot when being actuated (a), model of the robot when unactuated (b).*

8. What engineering work is being done?

- A novel actuator is being designed, and eventually a robot with dynamics similar to Fig. 3 will be constructed.
- The actuator is named the Variable Stiffness Series Elastic Actuator (VSSEA), because it is essentially similar to the standard Series Elastic Actuator (SEA) design of Robinson, Pratt-Willimason, but with the additional capability of variable stiffness. It uses two antagonistic nonlinear (quadratic) springs to effectively create a linear spring.

9. Why is this actuator special?

- In incorporates an energy storage mechanism (springs), variable compliance, and force control in a single package.
- Without any mechanical changes, and assuming that the motors are non-back-drivable, a robot built with the VSSEA design(Fig. 4a) could become a purely dynamic walker (Fig.4b) simply by removing power from the actuators.
- Another benefit of this topology is that control and analysis of the system are very close to linear, provided the stiffness is not changed quickly during operation, since the actuator model reduces to Fig.4d if the precompression motor is kept a constant length.

10. Why are springs needed?

- By measuring the deflection of the spring, you can accurately compute the force on the output mass.
- Springs reduce the output impedance of the actuator. Low output impedance is beneficial for force control.
- They provide a passive energy storage device.

11. Why is variable stiffness needed?

- Based upon other researchers' experience with real-world passive-dynamic bipedal robots, it seems common for the limit cycle of passive-dynamic robots to be relatively small, making their operation hard to reliably demonstrate by hand. It seems likely that despite accurate numerical simulations, differences between simulation and reality will be significant enough that some tuning may be required. Including a variable stiffness spring gives enough design flexibility to efficiently explore experimentally the range of the passive-dynamic limit cycle under realistic conditions.

12. How is this actuator different from other variable-stiffness actuators?

- The difference in topology can be seen in Fig. 4a and Fig. 4b.
- Most researchers working on variable-stiffness mechanisms, use actuators and quadratic springs in a different component topology than the VSSEA. They follow the principle that actuating antagonistic motors in common mode changes stiffness, and differential actuation changes position. In the VSSEA design, the stiffness and position are actuated independently.
- The reason for the different topology is due to a different goal. Most researchers desire for the stiffness of the actuator to be varied quickly and continuously throughout a motion, and use this stiffness change as an essential part of the control system (for, say, the purposes of safety, or for bio-mimetic reasons. In contrast to this, when using the VSSEA actuator in passive-dynamic robot models such as the one presented in this paper, rather than *vary* the effective stiffness, we desire to *hold the stiffness close to some optimal value* corresponding to some stable, efficient limit cycle, while still being able to add energy to the system via actuation.

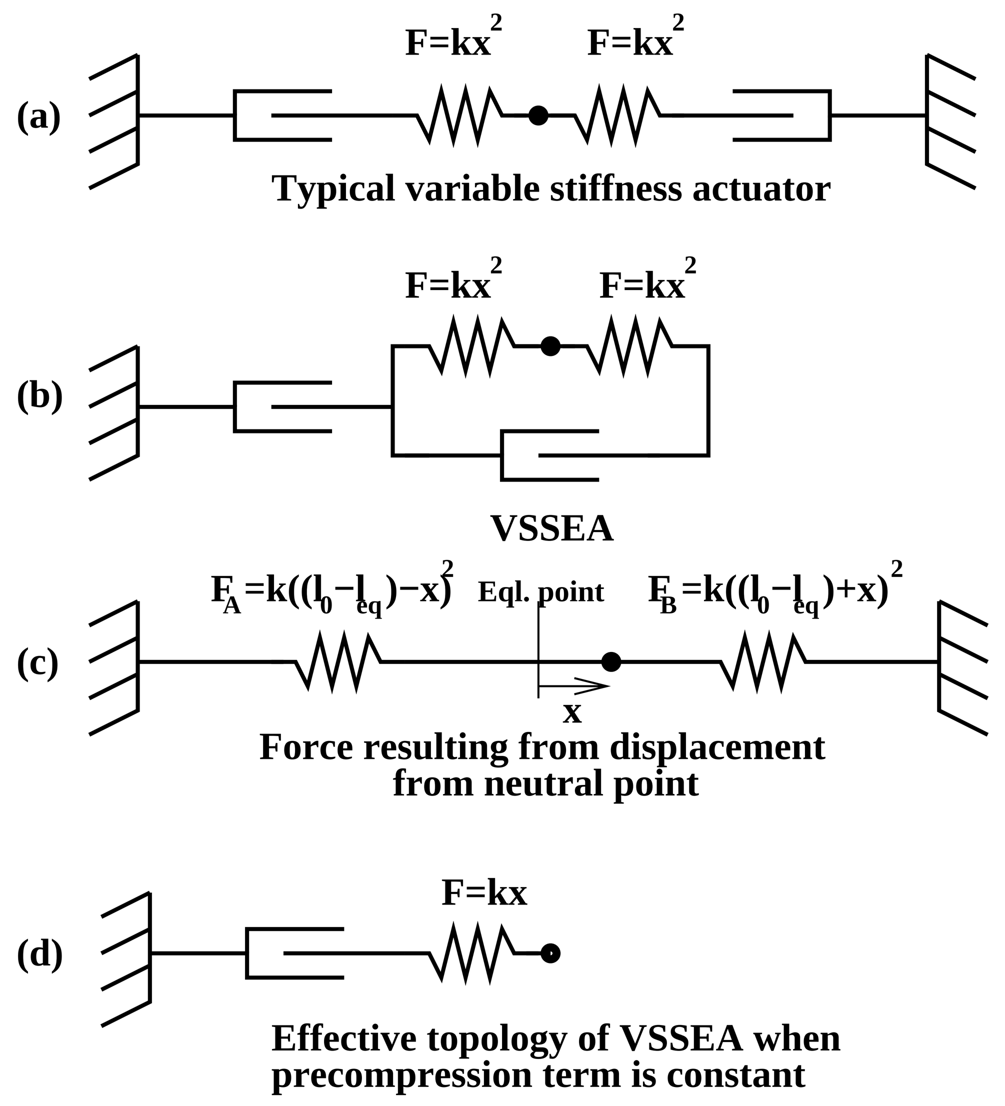

Figure 4: *Comparison of typical variable stiffness topology (a), VSSEA topology (b), the force on the output mass when both actuators are locked (c), and the effective VSSEA topology when only the precompression actuator is locked (d).*

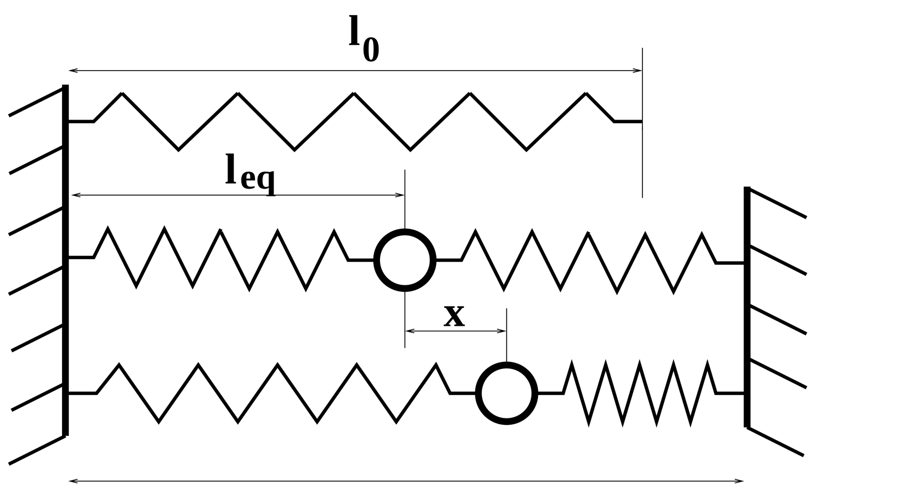

Figure 5: *Mechanical configuration of two compression springs used in the VSSEA, emphasizing spring lengths.*

13. Why are nonlinear springs needed?

- Refer to Fig. 4c and Fig. 5 for relevant measurements.
- Assume we have an output mass sandwiched between two antagonistic compression springs A and B. Let x be the displacement of the mass from the equilibrium point. Here l_0 is the natural length of the spring, and l_{eq} is the length of the spring when in equilibrium with the antagonistic spring (at no-load conditions on the output mass). We assume that the springs are always under compression, (i.e. that $l_0 \geq l_{eq}, x \leq l_0 - l_{eq}$).
  If the force-distance relationship for each spring when displaced from its natural length l_0 by a distance d is equal to
  $$F(x) = K_l d$$
  where K_l is the linear stiffness constant for that spring in units of N/m, then the force on the output mass is
  $$\begin{aligned} F &= F_A - F_B \\ &= K_l[(l_0 - l_{eq}) - x] - K_l[(l_0 - l_{eq}) + x] \\ &= -2K_l x \end{aligned}$$
  The pretension term $(l_0 - l_{eq})$ has no effect on the force the output mass sees, so a system which uses linear springs is not adjustable.
  However, if the reactive force from the spring when displaced from its natural length l_0 by a distance d is equal to
  $$F = K_n d^2$$
  where K_n is the nonlinear stiffness constant for that spring in units of N/m^2, then the force on the output mass is
  $$\begin{aligned} F &= F_A - F_B \\ &= K_n[(l_0 - l_{eq}) - x]^2 - K_n[(l_0 - l_{eq}) + x]^2 \\ &= K_n[l_0 - l_{eq} - x][l_0 - l_{eq} - x] \\ &\quad - K_n[l_0 - l_{eq} + x][l_0 - l_{eq} + x] \\ &= K_n[l_0^2 - 2l_0 l_{eq} - 2l_0 x + l_{eq}^2 + 2l_{eq}x + x^2] \\ &\quad - K_n[l_0^2 - 2l_0 l_{eq} + 2l_0 x + l_{eq}^2 - 2l_{eq}x + x^2] \\ &= K_n[-4l_0 x + 4l_{eq}x] \\ &= -4K_n[l_0 - l_{eq}]x \end{aligned}$$
  Notice that the effective stiffness K_{eff}, can be adjusted through the equilibrium precompression term $(l_0 - l_{eq})$.
  We now define $K_{eff} = -4K_n(l_0 - l_{eq})$ to be the effective linear spring constant in units of N/m. This gives us $F = K_{eff}x$, which is a linear spring that obeys Hooke's Law.
  In summary, the combination of two quadratic rate springs results effectively in a linear spring with a variable stiffness.
- This phenomenon seems well known by many other researchers investigating variable-stiffness mechanisms. The maximum dynamic range of stiffness for two antagonistically paired quadratic springs is limited to a 2:1 ratio, but we believe this range to be sufficient for the purposes of tuning the dynamics of a passive-dynamic robot.

14. Mechanical Description

A CAD representation of the actuator is in shown in Fig. 6 and Fig. 7. A prototype has been manufactured and is under testing. The mechanical properties of the prototype are listed in Table 1.

Table 1: *Parameters of Prototype Actuator*

Parameter	Value	Units
Max length	89	cm
Overall mass	4.5	kg
Spring stroke	20	cm
No-force stroke	$>$10	cm
Max. K_{eff}	6400	N/m
Min. K_{eff}	3200	N/m
Main Motor	90	W
Precompression Motor	5	W
Max. Force	$>$320	N

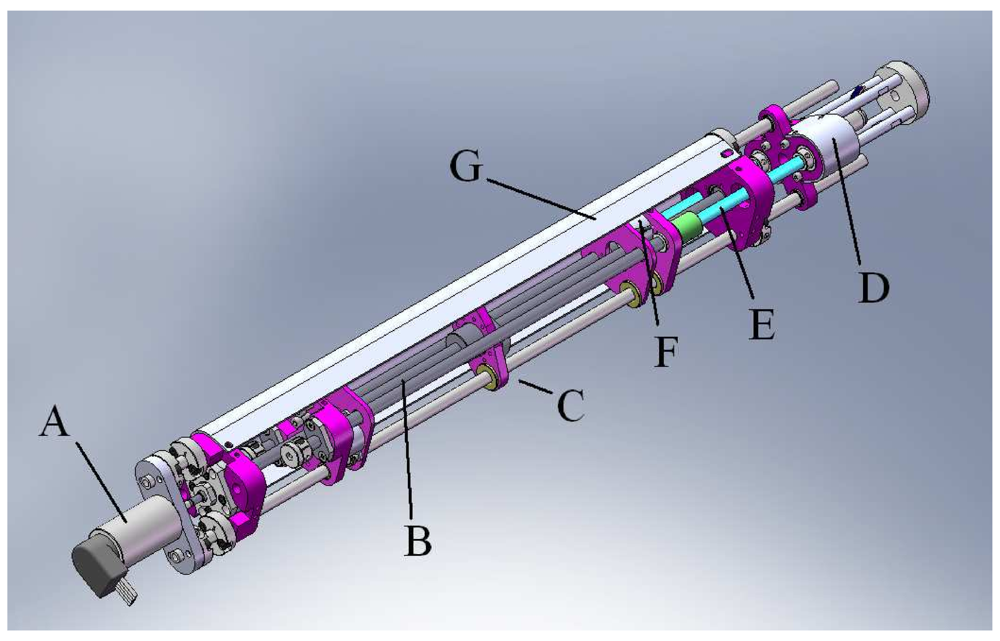

Figure 6: *CAD rendering of the VSSEA.*

At the heart of the actuator is the "main" motor (A) connected to a ballscrew (B). The ballscrew nut (C) is sandwiched between two variable-rate quadratic springs (not shown), which themselves are put under some variable amount of precompression by what we will call the "precompression" motor (D) and leadscrews(E). Two load cells (F) measure the compressive force acting on each spring. To determine the position of the output link (connected at D), length sensors (not shown) measure the length of each spring. The length sensors can also be used to double check the accuracy of the forces measured by the load cells, because the force-compression characteristics of the spring are known. A dust cover (G) keeps the actuator interior clean.

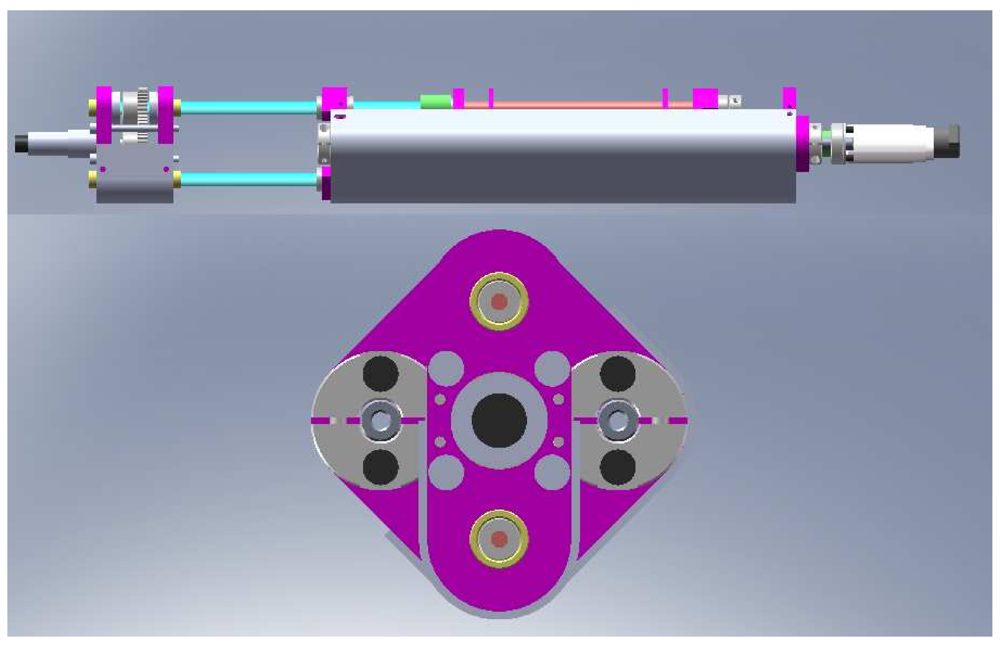

Figure 7: *Another view of the VSSEA.*

The prototype is composed mainly of aluminum, steel, and cast magnesium parts, but is not yet as lightweight as desired. Force-control is accomplished via a simple PD control rule implemented using feedback from quadrature encoders. Photos of the prototype are shown during presentation.

15. Conclusion and Future Work

We have presented motivation for studying the rather complex model shown in Fig.3, and introduced the necessity and benefits of using a novel actuator called the VSSEA. The VSSEA prototype has been manufactured, and after testing, four such actuators will be used to implement the biped model. We have also argued that this biped model is likely capable of both walking and running modes of operation. While the equations of motion corresponding to the biped model of Fig. 3 have been derived, a complete simulation including impact modeling and system stability has not yet been completed. In the future, we plan to study the limit cycle stability of the model, present prototype actuator test results, and examine experimental measurements of stability of a robot built with VSSEAs. It is hoped that the experimental results will verify the practicality of exploiting passive-dynamic limit cycles for energetically efficient legged locomotion.