Quantifying Gait Robustness of Passive Dynamic Robots

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Abstract: How can we quantify the robustness of the gait of a passive-dynamic robot? This paper proposes a simulation-based measurement of the robustness of a passive-dynamic system. Specifically, the measure we propose is the minimum distance from any point on the limit cycle to any unstable state, measured in the norm of the gradient of the Lagrangian. Simulation results and their limitations are presented.

1. Introduction

The stability and robustness of passive-dynamic robots has traditionally been examined via eigenvalue analysis of linearized models [3]. Although linear analysis is relatively straightforward to derive, this approximation is only valid for states close to the equilibrium point, and may mislead us about the large-disturbance stability of the robot, where nonlinearity may dominate the motion.

In contrast to the traditional approach, we investigate the nonlinear equations of motion via simulation. Rather than look at the system stability near the limit cycle, we instead investigate how far one can safely go away from the limit cycle in any direction and still return to it. Therefore, we call this distance the NR (Non-Returning) radius, the minimum distance from any point on the limit cycle to any non-returning state.

2. Passive Dynamic Systems Studied

In this paper, we examine the robustness of the the compass biped and biped with semicircular feet, as shown in Fig. 1a and 1b. Derivations of the equations of motions of these systems can be found in [1][2].

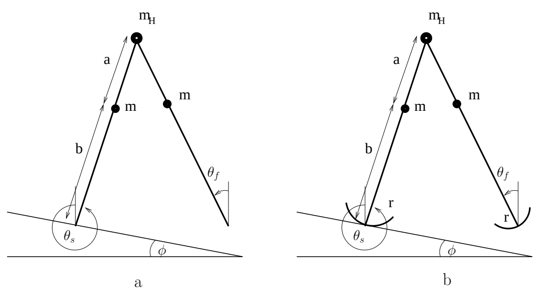

Fig.1 **a**. Compass Biped model. **b**. Compass biped with semicircular feet.

3. Visualization of Limit Cycles

Although the compass biped has a four dimensional state space $(\dot{\theta}_f, \dot{\theta}_s, \theta_f, \theta_s)$ which is hard to visualize, we can take a cross-section of this space by specifying the position of the robot to be the instant right after collision. This gives us the plot shown in Fig. 2. Stable areas are yellow, indicating 10 or more seconds of walking if started with the angular velocities specified at that point. Unstable areas are black or purple.

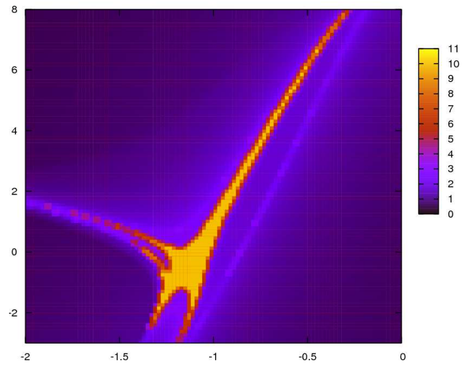

Fig.2 A 2D cross-section of the 4D basin of attraction for a compass biped, at the instant just after heelstrike collision. X-Y axes are angular velocities $\dot{\theta}_s, \dot{\theta}_f$.

If we plot the same graph again, but this time use $\frac{\partial L}{\partial \dot{\theta}_s}, \frac{\partial L}{\partial \dot{\theta}_f}$ as the X and Y axes, we get Fig. 3. Space limitations prevent the inclusion of many such slices here, but it is the opinion of the authors that this visualization provides better intuition into the limit cycle's attractive region because there are two important curves corresponding to the momentum of each leg.

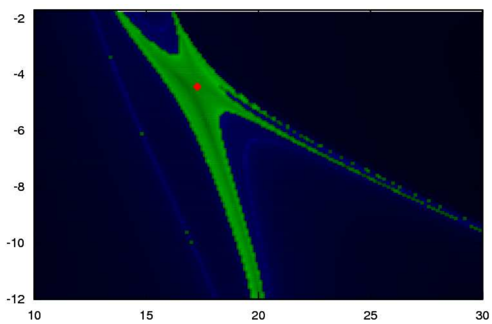

Fig.3 The same graph as Fig. 2, but here the X-Y axes are generalized momenta $\frac{\partial L}{\partial \dot{\theta}_s}, \frac{\partial L}{\partial \dot{\theta}_f}$. Green indicates a state that returns to the limit cycle, and darker green indicates it returns more quickly to the limit cycle. Blue states do not return to the limit cycle, and dark blue states indicate the states where the robot falls down more quickly. The red dot is where the limit cycle passes through this slice of the 4D space.

4. Definition of the NR Radius

We now define the NR (Non-Returning) radius of a system possessing a limit cycle.

In words, the NR radius is the minimum distance from any point on the limit cycle to any point outside the limit cycle's region of attraction. This is a fairly general definition, and the notion of "distance" could be defined mathematically by several different norms. Also, the domain the distance is measured in is also left unspecified, and could be for example the angular velocity domain, the generalized momenta domain, or some transformation of either. In this paper, we use the domain of ∇L, which corresponds to units of momentum and gives the following definition:

> The NR radius r_{NR} of a system with a Lagrangian L and generalized variables q is defined as
>
> $$r_{NR} = \min \|(\nabla L)_x - (\nabla L)_y\|_2 \,, \; x \in Q_{LC}, \; y \in Q_{NR}$$
>
> where $Q \subseteq R^n$ is the state space of the system (e.g. the set of all $q = (\dot{\theta}_f, \dot{\theta}_s, \theta_f, \theta_s)$. Here $Q_{LC} \subseteq Q$ is the set of states reperesenting the stable limit cycle, and $Q_{NR} \subseteq Q$ are states which do not return to the limit cycle. $(\nabla L)_x$ means the gradient evaluated at point x. The norm is Euclidian.

The graphical interpretation would be to imagine a sphere expanding in 4D from the limit cycle in Fig. 3, with the NR radius being the largest sphere which fits entirely in the green region.

The physical interpretation is that the robot's state at the limit cycle can be be disturbed once by the amount r_{NR}, at any instant in time, in any direction, and the robot will not fall over.

5. Simulation Results

Measurement of the NR radii for various ratios of hip mass m_H to leg mass m are shown in Fig. 4. Similarly, the NR radii for varius ratios of a and b are shown in Fig. 5.

Statistical methods used in the simulation resulted in a small amount of noise being introduced in the curves. Therefore, these curves are an upper bound; the exact values lie slightly below the curves (though probably not more than 0.001).

The simulation was written in LISP, using the Matlisp interface to LAPACK and the CL-OPENGL interface to OpenGL. All simulations were performed with a timestep of 0.001s using a Runge-Kutta 4/5 integrator. Heelstrike collision instants and were found with Newton-Raphson or secant zero-finding methods and are accurate to machine precision (32 bit), and total system energy variation error per step at the limit cycle is $< 10^{-13}$J.

Future simulations will measure the stability of bipeds with feet, knees, a torso link, or an interleg spring. It is expected that a biped with feet will have a larger NR radius.

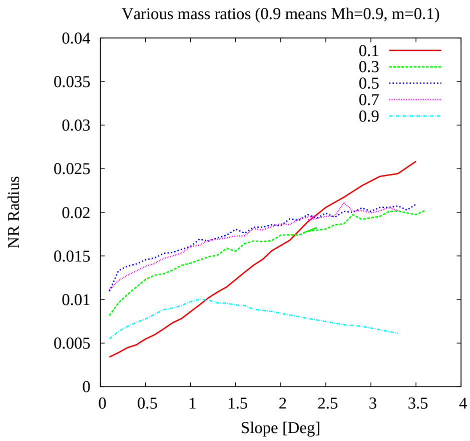

Fig.4 The NR radius when $a = 0.5$, $b = 0.5$ and the masses m_H and m are varied.

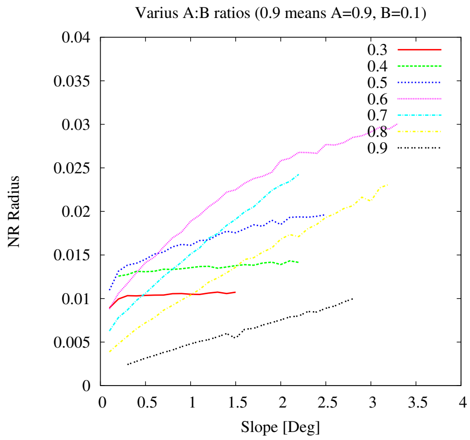

Fig.5 The NR radius when $m = 0.5$, $m_H = 0.5$ and the lengths a and b are varied.

6. Applicability and Limitations

The NR radius is a measurement applicable to not only bipedal walkers, but also to any dynamic system possessing a limit cycle or attractive point. Although difficult to solve for analytically, this quantity can be found via numerical simulation.

The primary limitation of this measurement is that it is dependent on how the coordinate system is defined. However, even with the limitation of coordinate variance, we are still able to answer questions like "How much more robust is this robot compared to this other robot?", assuming appropriately similar coordinate definitions.

References

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